

Q. / $N(t) = \frac{5t}{27+t^2}$

a) $N'(t) = \frac{5(27+t^2) - 5t(2t)}{(27+t^2)^2} = \frac{135 + 5t^2 - 10t^2}{(27+t^2)^2} = \frac{-5t^2 + 135}{(27+t^2)^2}$

$N''(t) = \frac{-10t(27+t^2)^2 - (-5t^2 + 135) \cdot 2(27+t^2) \cdot 2t}{(27+t^2)^4}$

$= \frac{-10t(27+t^2)^2 - 4t(-5t^2 + 135)(27+t^2)}{(27+t^2)^4}$

$= \frac{-2t(5(27+t^2) + 2(-5t^2 + 135))}{(27+t^2)^3}$

$= \frac{-2t(135 + 5t^2 - 10t^2 + 270)}{(27+t^2)^3} = \frac{-2t(-5t^2 + 405)}{(27+t^2)^3}$

b) epidemic worst: $\rightarrow N'(t) = 0$ and $N''(t) < 0$

$\rightarrow -5t^2 + 135 = 0 \quad N''(\sqrt{t}) < 0$

$5t^2 = 135$

$t^2 = \frac{135}{5}$

$t = \sqrt{27} \approx 5.2$

so $\boxed{t = 5.2 \text{ weeks}}$

c) $N(\sqrt{27}) = \frac{5\sqrt{27}}{54} = 0.481 \rightarrow \underline{48 \text{ new cases}}$

d) $N''(t) = 0 \Rightarrow -2t = 0$ or $-5t^2 + 405 = 0$

two
repeated

$t^2 = \frac{405}{5}$

$t = \sqrt{81} = \underline{9 \text{ weeks}}$

$N(9) = \frac{5 \cdot 9}{27+81}$

$= 0.417 \rightarrow \underline{42 \text{ cases}}$

$$e) \lim_{t \rightarrow \infty} \frac{5t}{27+t^2} = \lim_{t \rightarrow \infty} \frac{5}{t} = \underline{0.}$$

$$Q_2/ \quad f(x) = ax^2 + bx + c$$

$$f(0) = 1 \quad f'(-1) = 8 \quad f''(x) = 4.$$

$$f(0) = \boxed{c=1}$$

$$f'(x) = 2ax + b \Rightarrow f'(-1) = -2a + b = 8 \quad \leadsto -2 \cdot 2 + b = 8$$

$$f''(x) = 2a = 4 \Rightarrow a = \frac{4}{2} \Rightarrow \boxed{a=2}$$

$$b = 8 + 4$$

$$\boxed{b=12}$$

$$Q_4/ \quad f(x) = \frac{3x^2 - 3x - 6}{2x^3 + 2x^2 - 12x}$$

$$a) \quad \text{V.A.: } 2x^3 + 2x^2 - 12x = 0 \Rightarrow 2x(x^2 + x - 6) = 0$$

$$\boxed{x=0}$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$\boxed{x=-3} \quad \text{u.} \quad \boxed{x=2}$$

$$b) \quad \text{H.A.: } \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{3}{2x} = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{3}{2x} = 0$$

$$\boxed{y=0}$$

$$Q_3/a) f(x) = \frac{(x+2)}{\sqrt{e^{(x^2-2x+3)^2}}}$$

$$= e^{\frac{(x^2-2x+3)^2}{x+2}}$$

$$f'(x) = e^{\frac{(x^2-2x+3)^2}{x+2}} \cdot \left(\frac{(x^2-2x+3)^2}{x+2} \right)'$$

$$= e^{\frac{(x^2-2x+3)^2}{x+2}} \cdot \frac{2(x^2-2x+3) \cdot (2x-2)(x+2) - (x^2-2x+3)^2 \cdot 1}{(x+2)^2}$$

$$f'(0) = e^{\frac{9}{2}} \cdot \frac{2 \cdot 3 \cdot (-2) \cdot 2 - 9}{2^2} = e^{4.5} \cdot \left(-\frac{33}{4} \right) = \underline{-742.6}$$

$$b) g(x) = (x^2-x+1)^{\sqrt{x^2-x+1}} = e^{\ln(x^2-x+1)^{\sqrt{x^2-x+1}}} = e^{\sqrt{x^2-x+1} \cdot \ln(x^2-x+1)}$$

$$g'(x) = e^{\sqrt{x^2-x+1} \cdot \ln(x^2-x+1)} \cdot \left(\sqrt{x^2-x+1} \cdot \ln(x^2-x+1) \right)'$$

$$= e^{\sqrt{x^2-x+1} \cdot \ln(x^2-x+1)} \cdot \left(\frac{2x-1}{2\sqrt{x^2-x+1}} \cdot \ln(x^2-x+1) + \sqrt{x^2-x+1} \cdot \frac{2x-1}{x^2-x+1} \right)$$

$$g'(0) = e^{1 \cdot \ln 1} \cdot \left(\frac{-1}{2} \cdot \ln 1 + 1 \cdot \frac{-1}{1} \right) = 1 \cdot (-1) = \underline{-1}$$

$$c) y - h(0) = h'(0)(x-0)$$

$$y - \frac{f(0)}{g(0)} = h'(0) \cdot x$$

$$y - e^3 = -762.7 \cdot x$$

$$\boxed{y = -762.7x + 20.1}$$

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$h'(0) = \frac{f'(0)g(0) - f(0)g'(0)}{(g(0))^2}$$

$$= \frac{-742.6 \cdot 1 - e^3}{1}$$

$$= -762.7$$